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Unit-free commuting π- regular Rings

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ABSTRACT

Let R be an associative ring with Unitary and U denoted the set of all invertible element of R. we say that if for every x, y $\in R\backslash U$ there exist a positive integer n and a$\in$R such that $(xy)^n= (yx)^n a (yx)^n$.
then R is a Unit-free commuting π-regular ring. we show that if R is a Unit-free commuting π-regular ring, then for any $e^2=e$, eRe is Unit-free commuting π-regular.
In this paper shown that the center c(R) of every Unit-free commuting π-regular ring R is again Unit-free commuting π-regular.

We also proved that evey Unit-free commutingπ-regular ring, then R is π-regular.

Keywords: *commuting π–regular rings, Unit–f ree commuting regular rings, reduced rings.*

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INTRODUCTION

Let R be a ring. R is called von Neumann regular ring if for each x∈R there exists a∈R such

that xax=x. Following [2], R is called π-regular ring if for any x∈R there exist a positive

integer n and a∈R such that $x^n ax^n=x^n$ Following [6], R is called a commuting regular ring

if for eachx, y∈R there exists a∈R such that xy=yxayx.

Following [4], R is called Unit-free commuting regular ring if for any x.y∈R\U there exist

a∈R such that xy=yxayx.

In recent years some authors have studied the Unit-free commuting regular rings [4].

We extend Unit-free commuting regular rings and in troduce the concept of Unit-free commuting π-regular rings as following:

Definition 1. R is called a Unit-free commuting π-regular if for every x, y∈R\U, there exist

a positive integern and a∈R such that $(xy)^n= (yx)^n a(yx)^n$

Proposition 1. The center c(R) of every Unit-free commuting π-regular ring R is again

Unit-free commuting π-regular.

Proof. Let x, y∈c(R). Then there exist a∈R and a positive integer n such that

$(xy)^n=(yx)^n a(yx)^n=(yx)^{2n}a=a\ (yx)^{2n}$

and so

$(xy)^n a=(yx)^{2n}a^2=a^2\ (yx)^{2n}$

Let $z=(yx)^n a^2$ then

$(yx)^n z(yx)^n=(yx)^{2n}a(yx)^n=(yx)^n a\ (yx)^n=(xy)^n$.

Now it is enough to show that $z\in c(R)$. First note that $(yx)^n a\in c(R)$, because for any $r\in R$

we have

$(yx)^n ar=ar(yx)^n=ar(yx)^n a(yx)^n=a(yx)^{2n}ra=(yx)^n ra=r\ (yx)^n a$

and so

$zr=((yx)^n a)ar=ar(yx)^n a=(yx)^n ara=r\ (yx)^n a^2=rz$

and the proof is complete.

Proposition 2. Let R be a Unit-free commuting π-regular ring. then for any $e^2=e$, eRe is

Unit-free commuting π-regular.

Proof. Let $x, y\in eRe$. Since R is Unit-free commuting π-regular we have $(xy)^n=(yx)^n a(yx)^n$

for some $a\in R$ and a positive integer n. Note that $(yx)^n=(yx)^n e=e(yx)^n$.

Thus $(xy)^n=(yx)^n eae\ (yx)^n$ and it follows that eRe is Unit-free commuting π-regular.

Proposition 3. Let R be a Unit-free commuting π-regular ring, then R, is π-regular.

Proof. It suffices to show that for every $x\in R\backslash U$ there exist an element $y\in R$ and an integer n such that $x^n=x^n yx^n$, because if x is invertible then $x=x^{(-1)}xx$. Assume that $x\in R\backslash U$, since R is Unit-free commuting π-regular, then there exists $y\in R$ such that $x^2=x^2yx^2$ and the proof is complete.

Remark 1. The ring of n×n matrices over a Unit-free commutingπ-regular ring is not nec-

essarily Unit-free commuting π-regular.

For example Z_2 is Unit-free commuting π-regular ring but $M_2(Z_2)$ is not.

Indeed, Let

$$x=\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$$

and

$$y=\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}$$

such that $x.y\in M_2(\mathbb{Z}_2)$
then

$$xy=(xy)^n=\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$$

and

$$yx=(yx)^n=\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}$$

for each n∈N If there exists $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\in M_2(\mathbb{Z}_2)$ such that

$$\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}$$

then 1=0 which is a contradiction.

Proposition 4. Every homomorphic image of a Unit-free commuting π-regular ring is Unit-free

commuting π-regular.

Proof. Let R, S be rings and f:R→S be a ring epimorphism. suppose that R is Unit-free

commutingπ-regular and Let $\nu.\omega\in S$. since f is an epimorphism there exist $x, y\in R$ such that

$f(x)=\upsilon$, $f(y)=\omega$. Then since R is Unit-free commuting π-regular, there exist $a\in R$ and a positive integer n such that $(xy)^n=(yx)^n a(yx)^n$. It follows that

$(\upsilon\omega)^n=(f(xy)^n)$
$=f((yx)^n a(yx)^n)$
$=(wv)^n f(a)\ (wv)^n$

this completes the proof.

Corollary 1. Let R be a Unit-free commuting π-regular ring. If I is an ideal of R, then R\I

is Unit-free commuting π-regular.

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