World Journal of Environmental Biosciences

Available online at: www.environmentaljournals.org
Volume 1, Issue 1: 79-82

Open Access   Short Communication

A Study on Detrended Fluctuation Analysis and Lyapunov Exponent of Northeast Rainfall of Tamil Nadu

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Abstract:

In this paper, fractal dimensional analysis is used to investigate the Tamil Nadu rainfall dynamics. Here, the time series data of Tamil Nadu rainfall (annual, south west and northeast) has been analyzed using Detrended fluctuation analysis (DFA). For estimating the 110 years rainfall data, the scaling exponent analysis is used. The result shows a varying degree of persistence over shorter and longer time scales corresponding to distinct values of the DFA. Our studies suggest that DFA of the annual rainfall of Tamil Nadu is β=1 which indicates the 1/f noise, typical of systems in a self organized criticality (SOC), the southwest rainfall β=0.725 i.e., the time series is long-range correlated and for the northeast rainfall β=0.5 the time series corresponds to Gaussian white noise. Further, the Lyapunov exponent for the northeast rainfall of Tamil Nadu is analyzed. This paper estimates the various parameters like lyapunov exponent, maximum lyapunov characteristic exponent, lyapunov time for the northeast rainfall of Tamil Nadu. It is found that, the lyapunov exponent λ is positive, which implies that the orbit is unstable and chaotic, the maximum value of lyapunov exponent is 4.09 and this result verifies the DFA calculation. Therefore, no trend in amplitude can be discerned from the time series and hence, the northeast monsoon rainfall is unpredictable and chaotic.

Keywords: Detrended fluctuation analysis, criticality, Lyapunov exponent, chaotic.

1.0 Introduction:

Tamil Nadu, located in southeast peninsular India, receives the major part of its annual rainfall during the northeast monsoon season (the three-month period from October to December). Coastal Tamil Nadu receives about 60% of its annual rainfall and interior Tamil Nadu receives about 40-50% of annual rainfall during northeast monsoon (India Metrological department, 1973). In comparison with Indian summer monsoon, the Northeast monsoon is characterized by limited aerial extent and average lesser rainfall amount. During northeast monsoon season, Tamil Nadu generally receives rainfall due to the formation of tough or low, cyclonic circulation, easterly waves, low pressure area, depression and cyclonic storm over Bay of Bengal, because, the northeast monsoon season is the major rainy season. In view of the frequent failure of northeast monsoon rainfall over Tamil Nadu and the consequent water scarcity conditions, the vicissitudes of the rainfall of Tamil Nadu state has led to considerable and widespread interest among the public, farmers as well as government circles in recent years. This study, now, belongs to the newly emerging multi-disciplinary science of non linear dynamics and chaos (Mandelbrot, 1982). Orun, M. and Koçak, K. (2009) used Detrended fluctuation analysis (DFA) to calculate scaling exponent of daily mean temperature, daily maximum temperature, daily minimum temperature and daily temperature differences for 52 stations in Turkey.

The Detrended Fluctuation Analysis (DFA) technique was introduced to investigate long-range power-law correlations. J. Alvarez-Ramirez et al. (2008) proposed an extension of the R/S method to estimate the Hurst exponent of high dimensional fractals and also commented that, due to the simplicity in implementation, the DFA is now becoming a widely used method in physics and engineering. A. Sarkar and P. Barat (2005) investigated long time series of the rainfall records for All India and different regions of India and succeeded in finding evidence for power law distributions of the rainfall quantity. Peters et. al. (2002) has presented a power law behavior in the distribution of rainfall over at least four decades. On the other hand, fractal geometry can be described only by an algorithm (Paul Bourke, 1991). In this paper, an attempt is being made to calculate the scaling exponent using the Detrended Fluctuation Analysis (DFA) for the annual, southwest and northeast monsoon of Tamil Nadu.

2.0 Research Methodology:

The Detrended Fluctuation Analysis (DFA) technique is used to calculate the scaling exponent for the annual, southwest and northeast monsoon of Tamil Nadu. The first step in the Detrended Fluctuation Analysis (DFA) procedure is to calculate the time series *x*(*i*) of length N.

$$y(k) = \sum_{i=1}^{k} [\mathrm{x(i)} - \langle \mathrm{x} \rangle]$$

Here, x indicates the mean value of x(*i*) 's. Next, the profile *Y*(*k*) is divided into *N* [*N* / *n*] *n* which is the non overlapping segments of equal length *n* . In the next step, the local trend for each segment is calculated by a least square fit of the data. The y-coordinate of the fitted line is denoted by $Y(k)_n$. Then the Detrended time series for the segment duration 'n' as $Y_s(k) = Y(k) - Y_n(k)$. The root-mean square fluctuation of the original time series and the Detrended time series is calculated by

$$\mathrm{F(n)} = \left\{ \left[(1/N) \sum_{k=1}^{N} \left(Y(k) - Y_n(k) \right) \right]^2 \right\}^{\frac{1}{2}}$$

By repeating this calculation to all segment sizes, a relationship between *F*(*n*) and *n* is obtained. Finally the double logarithmic plot of *F*(*n*) versus *n* is used to calculate the slope, which gives the scaling exponent β. The value of the scaling component β is interpreted as follows.

(i) if $0 < \beta < 0.5$ then the time series is long-range anti-correlated
(ii) if $\beta > 0.5$ then the time series is long-range correlated
(iii) $\beta = 0.5$ corresponds to Gaussian white noise
(iv) While $\beta = 1$ indicates the $1/f$ noise, typical of systems in a Self Organized Criticality (SOC).

Consider two points in a space, X_0 and $X_0 + \Delta X_0$, each of which will generate an orbit in that space using some equation or system of equations. These orbits can be thought of as parametric functions of a variable, that is, something like time. If we use one of the orbits a reference orbit, then the separation between the two orbits will also be a function of time. Because sensitive dependence can arise only in some portions of a system (like the logistic equation), this separation is also a function of the location of the initial value and has the form $\Delta X(X_0, t)$. In a system with attracting fixed points or attracting periodic points, $\Delta X(X_0, t)$ diminishes asymptotically with time. If a system is unstable like pins balanced on their points, then the orbits diverge exponentially for a while, but eventually settle down. For chaotic points, the function $\Delta X(X_0, t)$ will behave erratically. It is thus useful to study the mean exponential rate of divergence of two initially close orbits using the formula

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \frac{|\Delta X(X_{0,} t)|}{|\Delta X_0|}$$

λ is useful for distinguishing among the various types of orbits. It works for discrete as well as continuous systems. The Lyapunov exponent for a set of "N" values can be found using the formula (Pietgen et al., 1992)

$$\lambda_k = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \ln \frac{|\Delta \mathrm{X}_{\mathrm{K}+1}|}{|\Delta \mathrm{X}_{\mathrm{K}}|}$$

The Lyapunov spectrum can be used to give an estimation of the fractal dimension of the considered dynamical system, i.e., the rainfall pattern.

$\lambda < 0$ The orbit attracts to a stable fixed point or stable periodic orbit. Negative Lyapunov exponents are characteristic of dissipative or non-conservative systems.

$\lambda = 0$ The orbit is a neutral fixed point (or an eventually fixed point). A Lyapunov exponent of zero indicates that the system is in some sort of steady state mode.

$\lambda > 0$ The orbit is unstable and chaotic. Nearby points, no matter how close, will diverge to any arbitrary separation. All neighborhoods in the phase space will eventually be visited. These points are said to be unstable.

2.1 Data:

For the study 110 years (1901-2010), data of the Annual, southwest and northeast monsoon rainfall over Tamil Nadu is considered. The above data is obtained from the Regional Meteorological Centre, Chennai.

3.0 Results:

Our studies suggest that DFA of the annual rainfall of Tamil Nadu is β=1(Fig 1) which indicates the 1/f noise, typical of systems in a self organized criticality (SOC), the southwest rainfall β=0.725 (Fig 2) i.e., the time series is long-range correlated and for the northeast rainfall β=0.5 (Fig 3) the time series corresponds to Gaussian white noise. Considering the chaotic behavior of the atmospheric, data the northeast rainfall of Tamil Nadu are calculated. From the respective error factors, the lyapunov exponents for 110 years are calculated.

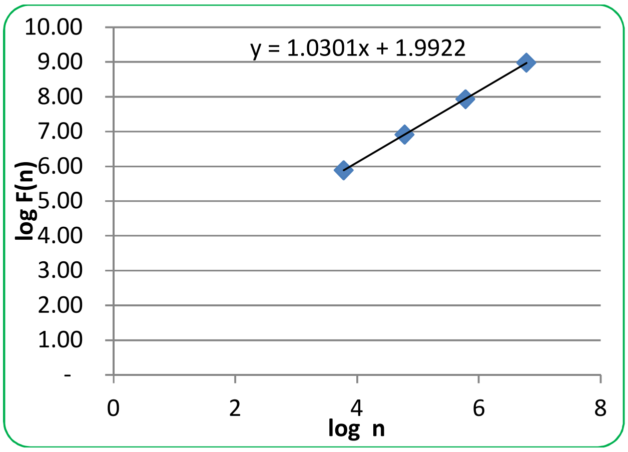

Fig 1: illustrates the Detrended fluctuation analysis of the annual rainfall of Tamil Nadu

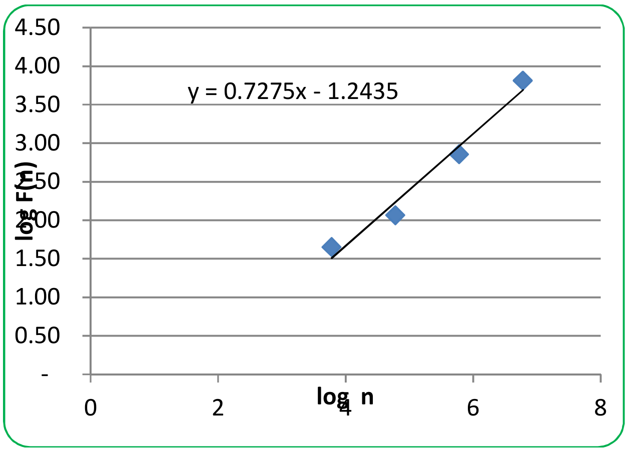

Fig 2: illustrates the Detrended fluctuation analysis of the southwest rainfall of Tamil Nadu

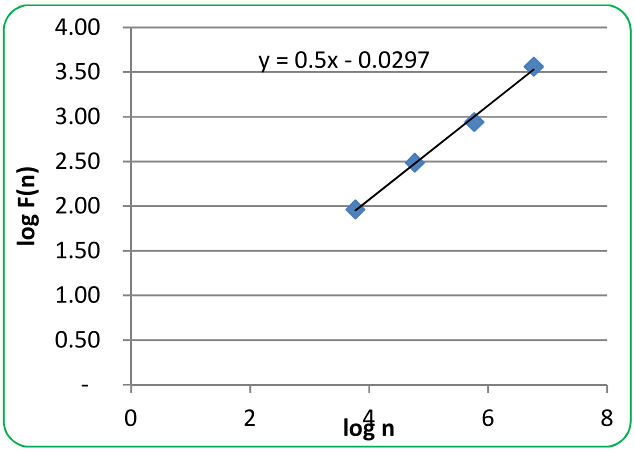

Fig 3: illustrates the Detrended fluctuation analysis of the northeast rainfall of Tamil Nadu

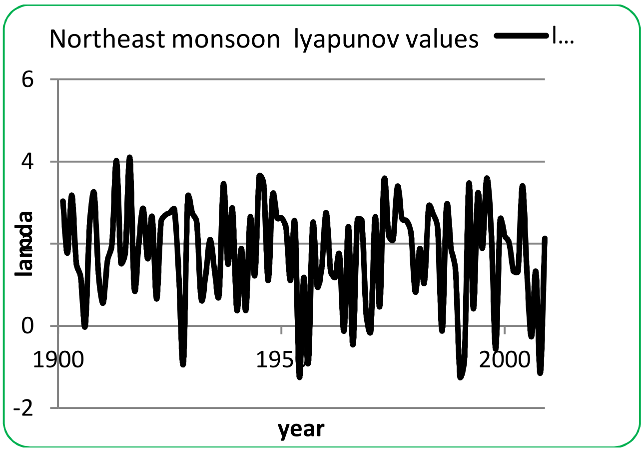

Fig 4: The lyapunov exponent λ values for the northeast monsoon rainfall Tamil Nadu

As the variation rainfall is cyclic, an attempt is made to calculate the lyapunov exponents for northeast monsoon rainfall after calculating λ (Fig 4). The λ value is positive (λ >0) which implies that the orbit is unstable and chaotic. The maximum value of lyapunov exponent for the above system is 4.09. This implies that the system is unstable and chaotic. The lyapunov time is defined as the inverse of the maximum lyapunov exponent. For the above system, the value of lyapunov time is 0.244. This indicates the time, after which, the system starts to behave in a chaotic manner.

4.0 References:

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